# Meng 356

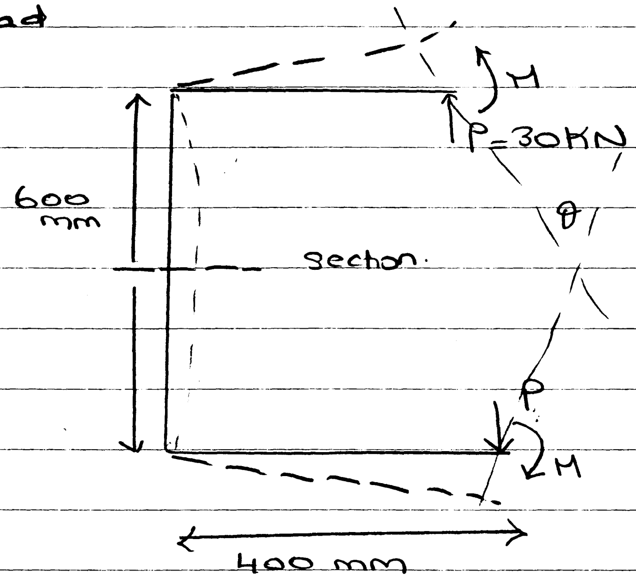
## Assignment # 2

Drill

Capacity = 30 KN

Design for strength & rigidity

$\theta \leqslant 0.01$ rad




Cast Iron (Brittle Material)

Tensile Strength = 290 Mpa

Compressive Strength = 965 Mpa

For Brittle Materials ⇒ F.S ≈ 4

$$\sigma_x \text{ tension} = \frac{N}{A} + \frac{M_z y_t}{I_z} \leqslant \frac{290}{FS}$$

Min

$$\sigma_x \text{ compression} = \frac{M_z y_c}{I_t} - \frac{N}{A} \leqslant \frac{965}{FS}$$

$$\theta = \frac{\partial y}{\partial M} \leqslant 0.01 \text{ rad}$$

For $\theta$ ⇒ using Castigliano's Theorem

↳ at point of Force & Moment

∴ assume a dummy Moment $M_I$

$$U = \int \frac{M^2}{2EI_z}dx \quad \text{over whole structure} + \int \frac{P^2}{2EA}dx$$

⇒ Total Energy of the Whole System

$$U = 2\int_0^{0.4} \frac{(Px + M_I)^2}{2EI_z}dx + \int_0^{0.6} \frac{(P(0.4)+M_I)^2}{2EI}dx + \int_0^{600} \frac{P^2}{2EA}dx$$

V.small neglected

$$\theta = \left.\frac{\partial U}{\partial M_I}\right|_{M_I=0} \Rightarrow \text{Castigliano's Theorem}$$

Differentiate first & then Integrate

$$\theta = \frac{2}{EI_z}\int_0^{0.4}(Px + M_I)\,dx + \frac{1}{EI_z}\int_0^{0.6}(0.4P + M_I)\,dx + 0$$

$$= \frac{2}{EI_z}\left[\frac{Px^2}{2} + M_I x\right]_0^{0.4} + \frac{1}{EI_z}\left[0.4Px + M_I x\right]_0^{0.6}$$

at $M_I = 0$

$$\theta = \frac{\cancel{2}}{EI_z}\left(\frac{Px^2}{\cancel{2}}\right)\Bigg|_0^{0.4} + \frac{1}{EI_z}\left[0.4Px\right]\Bigg|_0^{0.6} \leq 0.01$$

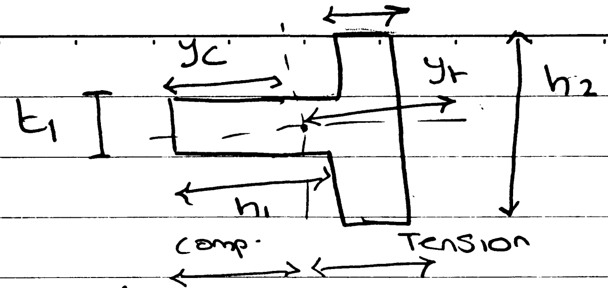
$$= \frac{Px^2}{EI_z}\Bigg|_0^{0.4} + \frac{1}{EI_z}(0.4Px)\Big|_0^{0.6}$$

$$= \frac{0.16P}{EI_z} + \frac{0.24P}{EI_z}$$

$$= \frac{0.4P}{EI_z} \leq 0.01 \quad \Rightarrow \text{Constraint}^{\text{Min}}$$

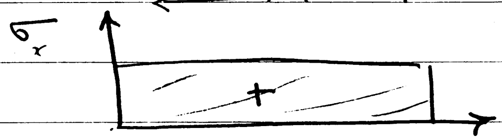
$$\Rightarrow \frac{P}{EI_z} \leq 0.025 \qquad P = 30\,\text{KN}$$

DATE

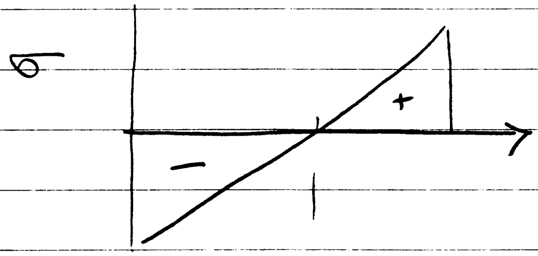



$\Rightarrow$ get $A$ $y_t$

$I_z$ $y_c$

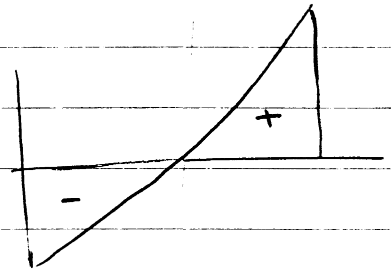



due to N




due to $M_z$

$\Rightarrow$




$\Rightarrow$ Resultant $\sigma$

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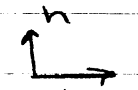
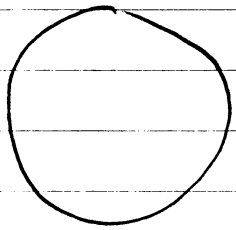
To get Moment of inertias

For a rectangle

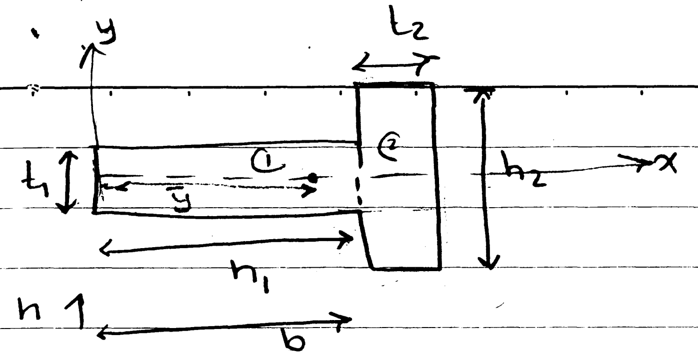



$\frac{1}{12} b^3 h$

For a Circle



$I = \frac{1}{4} \pi r^4$




$t_1 = t_2$
$h_1 = h_2$ } one assumption

To calculate Centre of gravity

| Area | $\bar{y}$ | $A\bar{y}$ |
|---|---|---|
| $t_1 h_1$ | $h_1/2$ | $\frac{t_1 h_1^2}{2}$ |
| $t_1 h_2$ | $h_1 + \frac{t_1}{2}$ | $t_1 h_2 (h_1 + \frac{t_1}{2})$ |
| $\Sigma A$ | | $\Sigma A\bar{y}$ |

$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

⇒ From this $\bar{y}$ can determine $\bar{y}_c$ & $\bar{y}_t$

To Calculate $I$

$I = \frac{1}{12} bh^3$ Divide area into 2 rectangles

Get $I_1$ & $I_2$ $I_{composite} = I_1 + I_2$
(whole area)

To get $I_1$ at its centroid (1)

$I_1 = \frac{1}{12} h_1 t_1^3$ (about its centroid)

About centroid of whole shape

$I_1' = I_1 + A_1 d_1^2$ d is the distance between the two axes

Repeat for $I_2$ ⇒ $I_2' = I_2 + A_2 d_2^2$

Then $I = I_1' + I_2'$

For ◎ $I = \frac{\pi}{64} (D^4 - d^4)$

ie $\sigma_T = \sigma_C$

For Steel: Tensile Strength = Compressive Strength

⇒ use Yield Strength F.S = 2